

AMENDMENTS TO THE CLAIMS

This listing of claims will replace all prior versions, and listings, of claims in the application:

Listing of Claims:

- 1 1. (Currently amended) A method for bounding the solution set of a
2 system of linear equations $\mathbf{Ax} = \mathbf{b}$, wherein \mathbf{A} is an interval matrix and \mathbf{b} is an
3 interval vector, the method comprising:
4 receiving the system of linear equations $\mathbf{Ax} = \mathbf{b}$;
5 storing $\mathbf{Ax} = \mathbf{b}$ in a memory in a computer system;
6 preconditioning the set of linear equations $\mathbf{Ax} = \mathbf{b}$ by multiplying through
7 both side of the linear equations by a matrix \mathbf{B} to produce a preconditioned set of
8 linear equations $\mathbf{BAx}=\mathbf{Bb}$, wherein the set of linear equations is a representation
9 of a global optimization problem;
10 substituting $\mathbf{M}_0 = \mathbf{BA}$ and $\mathbf{r} = \mathbf{Bb}$ to produce $\mathbf{M}_0\mathbf{x} = \mathbf{r}$;
11 widening the matrix \mathbf{M}_0 to produce a widened matrix \mathbf{M} , wherein the
12 midpoints of the interval elements of \mathbf{M} form the identity matrix; and
13 using \mathbf{M} and \mathbf{r} to compute ~~the hull~~ a hull \mathbf{h} of the system $\mathbf{Mx} = \mathbf{r}$, which
14 bounds the solution set of the system $\mathbf{M}_0\mathbf{x} = \mathbf{r}$;
15 wherein the interval operations involved in bounding the solution set are
16 performed using a special-purpose interval arithmetic unit configured to perform
17 interval arithmetic operations.

- 1 2. (Previously presented) The method of claim 1, wherein the method
2 further comprises computing the matrix \mathbf{B} by:

3 computing an approximate center \mathbf{A}_C of the interval elements of matrix \mathbf{A} ;
4 and
5 forming \mathbf{B} by computing an approximate inverse of \mathbf{A}_C , $\mathbf{B} = (\mathbf{A}_C)^{-1}$.

1 3 (Canceled).

1 4. (Previously presented) The method of claim 1, further comprising
2 assuring that $\sup(r_i) \geq 0$ by changing the sign of r_i ~~[[()]]~~ and x_i ~~[[()]]~~ if necessary,
3 wherein r_i is an element of \mathbf{r} .

1 5. (Original) The method of claim 1, further comprising:
2 determining if \mathbf{M} is regular; and
3 using the Gauss-Seidel process for computing the hull \mathbf{h} if \mathbf{M} is not
4 regular.

1 6. (Currently amended d) A computer-readable storage medium storing
2 instructions that when executed by a computer cause the computer to perform a
3 method for bounding the solution set of a system of linear equations $\mathbf{Ax} = \mathbf{b}$,
4 wherein \mathbf{A} is an interval matrix and \mathbf{b} is an interval vector, the method
5 comprising:
6 receiving the system of linear equations $\mathbf{Ax} = \mathbf{b}$;
7 storing $\mathbf{Ax} = \mathbf{b}$ in a memory in a computer system;
8 preconditioning the set of linear equations $\mathbf{Ax} = \mathbf{b}$ by multiplying through
9 both side of the linear equations by a matrix \mathbf{B} to produce a preconditioned set of
10 linear equations $\mathbf{BAx} = \mathbf{Bb}$, wherein the set of linear equations is a representation
11 of a global optimization problem;
12 substituting $\mathbf{M}_0 = \mathbf{BA}$ and $\mathbf{r} = \mathbf{Bb}$ to produce $\mathbf{M}_0\mathbf{x} = \mathbf{r}$;

13 widening the matrix \mathbf{M}_0 to produce a widened matrix \mathbf{M} , wherein the
14 midpoints of the interval elements of \mathbf{M} form the identity matrix; and
15 using \mathbf{M} and \mathbf{r} to compute ~~the hull~~ a hull \mathbf{h} of the system $\mathbf{M}\mathbf{x} = \mathbf{r}$, which
16 bounds the solution set of the system $\mathbf{M}_0\mathbf{x} = \mathbf{r}_i$
17 wherein the interval operations involved in bounding the solution set are
18 performed using a special-purpose interval arithmetic unit configured to perform
19 interval arithmetic operations.

1 7. (Previously presented) The computer-readable storage medium of claim
2 6, wherein the method further comprises computing the matrix \mathbf{B} by:
3 computing an approximate center \mathbf{A}_C of the interval elements of matrix \mathbf{A} ;
4 and
5 forming \mathbf{B} by computing an approximate inverse of \mathbf{A}_C , $\mathbf{B} = (\mathbf{A}_C)^{-1}$.

1 8. (Previously presented) The computer-readable storage medium of claim
2 6, wherein using \mathbf{M} and \mathbf{r} to compute the hull \mathbf{h} involves:
3 forming \mathbf{P} as an inverse of the left endpoint of \mathbf{M} ;
4 forming $c_i = 1/(2P_{ii} - 1)$ for $i = 1, \dots, n$;
5 forming $z_i = (\inf(r_i) + \sup(r_i))P_{ii} - e_i^T \mathbf{P} \sup(\mathbf{r})$, wherein e_i^T is a unit vector in
6 which the i -th element is 1 and other elements are 0, and wherein r_i is an element
7 of \mathbf{r} ;
8 setting $\inf(h_i) = c_i z_i$ if $z_i > 0$;
9 setting $\inf(h_i) = z_i$ if $z_i \leq 0$; and
10 setting $\sup(\mathbf{h}) = \mathbf{P} \sup(\mathbf{r})$.

1 9. (Previously presented) The computer-readable storage medium of claim
2 6, wherein the method further comprises assuring that $\sup(r_i) \geq 0$ by changing the
3 sign of r_i $[[()]]$ and $x_i[[()]]$ if necessary, wherein r_i is an element of \mathbf{r} .

1 10. (Original) The computer-readable storage medium of claim 6, wherein
2 the method further comprises:
3 determining if \mathbf{M} is regular; and
4 using the Gauss-Seidel process for computing the hull \mathbf{h} if \mathbf{M} is not
5 regular.

1 11. (Currently amended) An apparatus that bounds the solution set of a
2 system of linear equations $\mathbf{Ax} = \mathbf{b}$, wherein \mathbf{A} is an interval matrix and \mathbf{b} is an
3 interval vector, comprising:
4 a receiving mechanism configured to receive the system of linear
5 equations $\mathbf{Ax} = \mathbf{b}$;
6 a special purpose arithmetic unit configured to perform interval
7 computations;
8 a storing mechanism configured to store $\mathbf{Ax} = \mathbf{b}$ in a memory in a
9 computer system;
10 a preconditioning mechanism within the special purpose arithmetic unit
11 that is configured to precondition the set of linear equations $\mathbf{Ax} = \mathbf{b}$ by
12 multiplying ~~through both side of the linear equations~~ by a matrix \mathbf{B} to produce a
13 preconditioned set of linear equations $\mathbf{BAx} = \mathbf{Bb}$, wherein the set of linear
14 equations is a representation of a global optimization problem;
15 a substituting mechanism within the special purpose arithmetic unit that is
16 configured to substitute $\mathbf{M}_0 = \mathbf{BA}$ and $\mathbf{r} = \mathbf{Bb}$ to produce $\mathbf{M}_0\mathbf{x} = \mathbf{r}$;
17 a widening mechanism within the special purpose arithmetic unit that is
18 configured to widen the matrix \mathbf{M}_0 to produce a widened matrix \mathbf{M} , wherein the
19 midpoints of the interval elements of \mathbf{M} form the identity matrix; and
20 a hull computing mechanism within the special purpose arithmetic unit
21 that is configured to use \mathbf{M} and \mathbf{r} to compute ~~the hull~~ a hull \mathbf{h} of the system
22 $\mathbf{Mx} = \mathbf{r}$, which bounds the solution set of the system $\mathbf{M}_0\mathbf{x} = \mathbf{r}$;

23 wherein the interval operations involved in bounding the solution set are
24 performed using the special-purpose interval arithmetic unit.

1 12. (Previously presented) The apparatus of claim 11, wherein the
2 preconditioning mechanism is configured to:
3 compute an approximate center A_C of the interval elements of matrix A ;
4 and to
5 form B by computing an approximate inverse of A_C , $B = (A_C)^{-1}$.

1 13. (Previously presented) The apparatus of claim 11, wherein the hull
2 computing mechanism is configured to:
3 form P as an inverse of the left endpoint of M ;
4 form $c_i = 1/(2P_{ii} - 1)$ for $i = 1, \dots, n$;
5 form $z_i = (\inf(r_i) + \sup(r_i))P_{ii} - e_i^T P \sup(r)$, wherein e_i^T is a unit vector in
6 which the i -th element is 1 and other elements are 0, and wherein r_i is an element
7 of r ;
8 set $\inf(h_i) = c_i z_i$ if $z_i > 0$;
9 set $\inf(h_i) = z_i$ if $z_i \leq 0$; and to
10 set $\sup(h) = P \sup(r)$.

1 14. (Previously presented) The apparatus of claim 11, wherein the
2 preconditioning mechanism is configured to assure that $\sup(r_i) \geq 0$ by changing
3 the sign of r_i $[[()]]$ and $x_i[[()]]$ if necessary, wherein r_i is an element of r .

1 15. (Original) The apparatus of claim 11, wherein the preconditioning
2 mechanism is configured to:
3 determine if M is regular; and to
4 terminate the process of computing the hull h if M is not regular.

1 16. (Currently amended) A method for bounding the solution set of a
2 system of linear equations $\mathbf{Ax} = \mathbf{b}$ by multiplying ~~through both side of the linear~~
3 equations by the matrix \mathbf{B} to produce a preconditioned set of linear equations
4 $\mathbf{BAx}=\mathbf{Bb}$, wherein the set of linear equations is a representation of a global
5 optimization problem, the method comprising:
6 receiving the system of linear equations $\mathbf{Ax} = \mathbf{b}$;
7 storing $\mathbf{Ax} = \mathbf{b}$ in a memory in a computer system;
8 substituting $\mathbf{M}_0 = \mathbf{BA}$ and $\mathbf{r} = \mathbf{Bb}$ producing $\mathbf{M}_0\mathbf{x} = \mathbf{r}$;
9 assuring that $\sup(r_i) \geq 0$ by changing the sign of r_i (and x_i) if necessary;
10 widening the matrix \mathbf{M}_0 to produce a widened matrix \mathbf{M} , wherein the
11 midpoints of the interval elements of \mathbf{M} form the identity matrix; and
12 using \mathbf{M} and \mathbf{r} to compute ~~the hull a hull~~ \mathbf{h} of the system $\mathbf{Mx} = \mathbf{r}$, which
13 bounds the solution set of the system $\mathbf{M}_0\mathbf{x} = \mathbf{r}$ by,
14 forming \mathbf{P} as an inverse of the left endpoint of \mathbf{M} ,
15 forming $c_i = 1/(2P_{ii} - 1)$ for $i = 1, \dots, n$,
16 forming $z_i = (\inf(r_i) + \sup(r_i))P_{ii} - e_i^T \mathbf{P} \sup(\mathbf{r})$,
17 wherein e_i^T is a unit vector in which the i -th element is 1
18 and other elements are 0, and wherein r_i is an element of \mathbf{r} ,
19 setting $\inf(h_i) = c_i z_i$ if $z_i > 0$,
20 setting $\inf(h_i) = z_i$ if $z_i \leq 0$, and
21 setting $\sup(\mathbf{h}) = \mathbf{P} \sup(\mathbf{r})$;
22 wherein the interval operations involved in bounding the solution set are
23 performed using a special-purpose interval arithmetic unit configured to perform
24 interval arithmetic operations.

1 17. (Original) The method of claim 16, further comprising:
2 determining if \mathbf{M} is regular; and

3 using the Gauss-Seidel process for computing the hull **h** if **M** is not
4 regular.

1 18. (Previously presented) The method of claim 16, wherein the method
2 further comprises computing the matrix **B** by:
3 computing an approximate center **A_C** of the interval elements of matrix **A**;
4 and
5 forming **B** by computing an approximate inverse of **A_C**, $\mathbf{B} = (\mathbf{A}_C)^{-1}$.